9. Forced Convection Correlations

Our primary objective is to determine heat transfer coefficients (local and average) for different flow geometries and this heat transfer coefficient (h) may be obtained by experimental or theoretical methods. Theoretical methods involve solution of the boundary layer equations to get the Nusselt number such as explained before in the previous lecture. On the other hand the experimental methods involve performing heat transfer measurement under controlled laboratory conditions and correlating data in dimensionless parameters. Many correlations for finding the convective heat transfer coefficient are based on experimental data which needs uncertainty analysis, although the experiments are performed under carefully controlled conditions. The causes of the uncertainty are many. Actual situations rarely conform completely to the experimental situations for which the correlations are applicable. Hence, one should not expect the actual value of the heat transfer coefficient to be within better than □10%of the predicted value.

9.1 Flow over Flat Plate

With a fluid flowing parallel to a flat plate we have several cases arise:

- Flows with or without pressure gradient
- Laminar or turbulent boundary layer
- Negligible or significant viscous dissipation (effect of frictional heating)

9.1.1 Flow with Zero Pressure Gradient and Negligible Viscous Dissipation

When the free-stream pressure is uniform, the free-stream velocity is also uniform. Whether the boundary layer is laminar or turbulent depends on the Reynolds number Re_X ($\rho u_\infty x/\mu$) and the shape of the solid at entrance. With a sharp edge at the leading edge the boundary layer is initially laminar but at some distance downstream there is a transition region and downstream of the transition region the boundary layer becomes turbulent. For engineering applications the transition region is usually neglected and it is assumed that the boundary layer becomes turbulent if the Reynolds number, Re_X, is greater than the critical Reynolds number, Re_{cr}. A typical value of $5\ x10^5$ for the critical Reynolds number is generally accepted.

The viscous dissipation and high-speed effects can be neglected if $Pr^{1/2}\,Ec<1$. The Eckert number Ec is defined as $Ec = u^2_\infty/C_p\,(T_S-T_\infty)$ with a rectangular plate of length L in the direction of the fluid flow.

9.1.1.1 Laminar Boundary Layer ($Re_x \leq 5x10^5$)

With heating or cooling starting from the leading edge the following correlations are recommended. Note: in all equations evaluate fluid properties at the film temperature (T_f) defined as the arithmetic mean of the surface and free-stream temperatures unless otherwise stated.

$$T_f= (T_S + T_\infty)/2$$

Flow with uniform surface temperature (T_S = constant)

- Local heat transfer coefficient

The Nusselt number based on the local convective heat transfer coefficient is expressed as

$$Nu_X = f_{\Pr} \operatorname{Re}_X^{1/2} \tag{9.1}$$

The expression of $f_{\Pr}$ depend on the fluid Prandtl number

For liquid metals with very low Prandtl number liquid metals ($\Pr \le 0.05$)

$$f_{\Pr} = 0.564 \Pr^{1/2}$$

For $0.6 < \Pr < 50$

$$f_{\Pr} = 0.332 \Pr^{1/3}$$

For very large Prandtl number

$$f_{\Pr} = 0.339 \Pr^{1/3}$$

For all Prandtl numbers; Correlations valid developed by Churchill (1976) and Rose (1979) are

$$Nu_x = \frac{0.3387 \operatorname{Re}_x^{1/2} \Pr^{1/3}}{\left[1 + \left(\dfrac{0.0468}{\Pr}\right)^{2/3}\right]^{1/4}} \tag{9.2}$$

- Average heat transfer coefficient

The average heat transfer coefficient can be evaluated by performing integration along the flat plate length, if Prandtl number is assumed constant along the plate, the integration yields to the following result:

$$\overline{Nu}_x = 2 Nu_x \tag{9.3}$$

Flow with uniform heat flux (q'' = constant)

- Local heat transfer coefficient

Churchill and Ozoe (1973) recommend the following single correlation for all Prandtl numbers

$$Nu_X = \frac{0.886 \operatorname{Re}_X^{1/2} \Pr^{1/2}}{\left[1 + \left(\dfrac{\Pr}{0.0207}\right)^{2/3}\right]^{1/4}} \tag{9.4}$$

9.1.1.2 Turbulent Boundary Layer ($Re_x > 5x10^5$)
With heating or cooling starting from the leading edge the following correlations are recommended.

- Local heat transfer coefficient

$Re_{cr} < Re_x \leq 10^7$ $$Nu_x = 0.0296 \, Re_x^{4/5} \, Pr^{1/3} \quad (9.5)$$

$Re_x > 10^7$ $$Nu_x = 1.596 \, Re_x (\ln Re_x)^{-2.584} \, Pr^{1/3} \quad (9.6)$$

Equation 9.6 is obtained by applying Colburn's j factor in conjunction with the friction factor suggested by Schlichting (1979).
With turbulent boundary layers the correlations for the local convective heat transfer coefficient can be used for both uniform surface temperature and uniform heat flux.

- Average heat transfer coefficient
If the boundary layer is initially laminar followed by a turbulent boundary layer some times called mixed boundary layer the Following correlations for $0.6 < Pr < 60$ are suggested:

$Re_{cr} < Re_x \leq 10^7$ $$\overline{Nu}_x = (0.037 \, Re_x^{4/5} - 871) \, Pr^{1/3} \quad (9.7)$$

$Re_x > 10^7$ $$\overline{Nu}_x = [1.963 \, Re_x (\ln Re_x)^{-2.584} - 871] \, Pr^{1/3} \quad (9.8)$$

9.1.1.3 Unheated Starting Length

Uniform Surface Temperature & Pr > 0.6
If the plate is not heated or (cooled) from the leading edge where the boundary layer develops from the leading edge until being heated at $x = x_o$ as shown in Figure 9.1, the correlations have to be modified.

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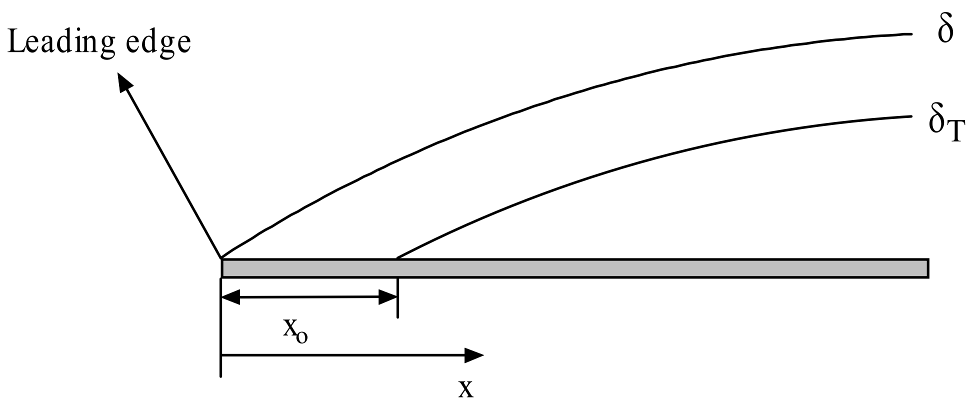


Figure 9.1 Heating starts at $x = x_o$

- Local heat transfer coefficient

Laminar flow ($Re_x < Re_{cr}$)

$$Nu_x = \frac{0.332\,Re_x^{1/2}\,Pr^{1/3}}{\left[1-\left(\frac{x_o}{x}\right)^{3/4}\right]^{1/3}} \tag{9.9}$$

Turbulent flow ($Re_x > Re_{cr}$)

$$Nu_x = \frac{0.0296\,Re_x^{4/5}\,Pr^{3/5}}{\left[1-\left(\frac{x_o}{x}\right)^{9/10}\right]^{1/9}} \tag{9.10}$$

- Average heat transfer coefficient over the Length (L – xo)

Laminar flow ($Re_L < Re_{cr}$)

$$\bar{h}_{L-x_o} = \left(\frac{k}{L-x_o}\right)0.664\,Re_L^{1/2}\,Pr^{1/3}\left[1-\left(\frac{x_o}{L}\right)^{3/4}\right]^{2/3} \tag{9.11}$$

$$= 2\left(\frac{h_{x=L}}{1-(x_o/L)}\right)\left(1-\left(\frac{x_o}{L}\right)^{3/4}\right)$$

Evaluate $h_{x=L}$ from Equation 9.9.

Turbulent flow ($Re_L > Re_{cr}$)

$$\bar{h}_{L-x_o} = \frac{0.037\,Re_L^{4/5}\,Pr^{3/5}\left[1-\left(\frac{x_o}{L}\right)^{9/10}\right]^{8/9} k}{L-x_o} \tag{9.12}$$

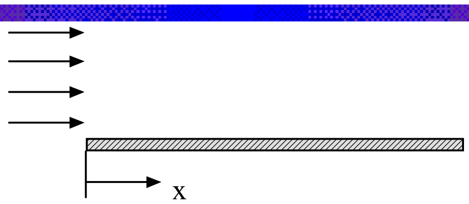
$$= 1.25\frac{1-\left(\frac{x_o}{L}\right)^{9/10}}{1-\left(\frac{x_o}{L}\right)} h_{x=L}$$

Evaluate $h_{x=L}$ from Equation 9.10.

Example 9.1: Experimental results obtained for heat transfer over a flat plate with zero pressure gradients yields

$$Nu_x = 0.04\,Re_x^{0.9}\,Pr^{1/3}$$

Where this correlation is based on x (the distance measured from the leading edge). Calculate the ratio of the average heat transfer coefficient to the local heat transfer coefficient.
Schematic:



Solution:
It is known that

$$Nu_x = \frac{h_x x}{k}$$

Local heat transfer coefficient

$$h_x = 0.04\frac{k}{x}\mathrm{Re}_x^{0.9}\ \mathrm{Pr}^{1/3}$$

The average heat transfer coefficient is:

$$\bar{h}_x = \frac{1}{x}\int_0^x h_x dx = \frac{0.04k\,\mathrm{Pr}^{1/3}}{x}\int_0^x \left(\frac{u_\infty x}{\nu}\right)^{0.9}\frac{1}{x}dx$$

$$= \frac{0.04k\,\mathrm{Pr}^{1/3}}{x}\left(\frac{u_\infty}{\nu}\right)^{0.9}\int_0^x x^{-0.1}dx$$

$$= \frac{0.04k\,\mathrm{Pr}^{1/3}}{0.9x}\left(\frac{u_\infty}{\nu}\right)^{0.9} x^{0.9}$$

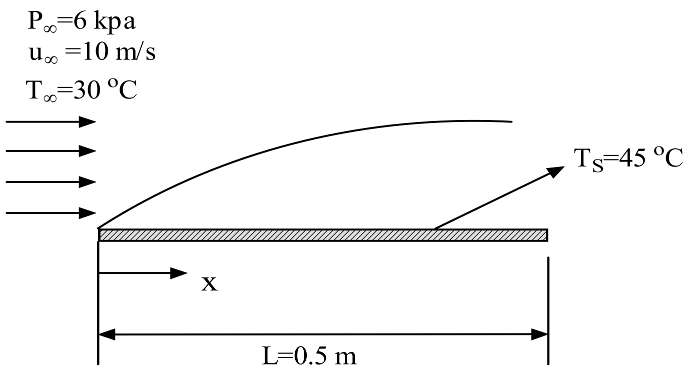
$$= \frac{0.04k}{0.9x}\mathrm{Re}_x^{0.9}\ \mathrm{Pr}^{1/3}$$

$$= \frac{h_x}{0.9}$$

The ratio of the average heat transfer coefficient to the local heat transfer coefficient is:

$$\frac{\bar{h}_x}{h_x} = \frac{1}{0.9}$$

Example 9.2: Air at a temperature of 30 ºC and 6 kPa pressure flow with a velocity of 10 m/s over a flat plate 0.5 m long. Estimate the heat transferred per unit width of the plate needed to maintain its surface at a temperature of 45 ºC.
Schematic:



Solution:

Properties of the air evaluated at the film temperature

$T_f = 75/2 = 37.5$ ºC

From air properties table at 37.5 ºC:

$\nu = 16.95 \times 10^{-6}$ m²/s.

k = 0.027 W/m. ºC

Pr= 0.7055

C_p= 1007.4 J/kg. ºC

Note: These properties are evaluated at atmospheric pressure, thus we must perform correction for the kinematic viscosity

$$\nu)_{act.} = \nu)_{atm.} x\ (1.0135/0.06) = 2.863\ x10^{-4}\ m^2/s.$$

Viscous effect check,

$$Ec = u^2_\infty / C_p(T_S - T_\infty) = (10)^2/(1007.4x15) = 6.6177x10^{-3}$$

It produce $Pr^{1/2}$ Ec<1 so that we neglect the viscous effect

Reynolds number check

$$Re_L = u_\infty L / \nu = 10 \times 0.5 / (2.863\ x10^{-4}) = 17464.2$$

The flow is Laminar because $Re_L \le 5x10^5$

Using the Equation 9.3

$$\overline{Nu}_L = 0.664\, Re_L^{1/2}\, Pr^{1/3}$$
$$= 0.664\ (17464.2)^{1/2}(0.7055)^{1/3}$$
$$= 78.12$$

Now the average heat transfer coefficient is

$$\overline{Nu}_L = \frac{\bar{h}L}{k}$$

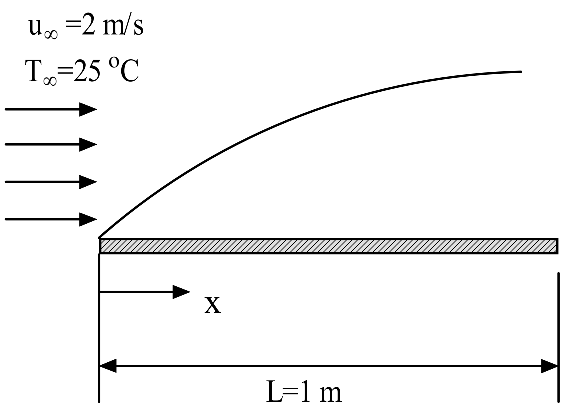
$$\bar{h} = 78.12x\ 0.027\ /0.5 = 4.22\ W/m^2.\ ºC$$

Then the total heat transfer per unit width equals

$$q = \bar{h}\, L\ (T_S - T_\infty) = 4.22 \times 0.5\ (45 - 30) = 31.64\ W/m$$

Example 9.3: Water at 25 ºC is in parallel flow over an isothermal, 1-m long flat plate with velocity of 2 m/s. Calculate the value of average heat transfer coefficient.

Schematic:



Solution:

Assumptions

- neglect the viscous effect
- The properties of water are evaluated at free stream temperature

From water properties table at 25 ºC:

$\nu = 8.57 \times 10^{-7}$ m/s.

k = 0.613W/m. ºC

Pr= 5.83

C_p= 4180 J/kg. ºC

Reynolds number check,

$$Re_L = u_\infty L / \nu = 2 \times 1 / (8.57 \times 10^{-7}) = 2.33 \times 10^6$$

The flow is mixed because $Re_L \geq 5 \times 10^5$

By using the Equation 9.7

$$\overline{Nu}_x = (0.037\,\mathrm{Re}_x^{4/5} - 871)\,\mathrm{Pr}^{1/3}$$
$$= [0.037 x (2.33 x 10^6)^{4/5} - 871](5.83)^{1/3}$$
$$= 6704.78$$
$$= \frac{\overline{h}_L L}{k}$$

The average heat transfer coefficient is

$$\overline{h}_L = 4110 \;\; \mathrm{W/m^2 \cdot K}$$

9.3 Flow over Cylinders, Spheres, and other Geometries

The flow over cylinders and spheres is of equal importance to the flow over flat plate, they are more complex due to boundary layer effect along the surface where the free stream velocity u_∞ brought to the rest at the forward stagnation point (u = 0 and maximum pressure) the pressure decrease with increasing x is a favorable pressure gradient (dp/dx<0) bring the pressure to minimum, then the pressure begin to increase with increasing x by adverse pressure gradient (dp/dx>0) on the rear of the cylinder. In general, the flow over cylinders and spheres may have a laminar boundary layer followed by a turbulent boundary layer.

The laminar flow is very weak to the adverse pressure gradient on the rear of cylinder so that the separation occurs at θ = 80º and caused wide wakes as show in Figure9.2.a.

The turbulent flow is more resistant so that the separation occurs at θ = 120º and caused narrow wakes as show in Figure9.2.b.

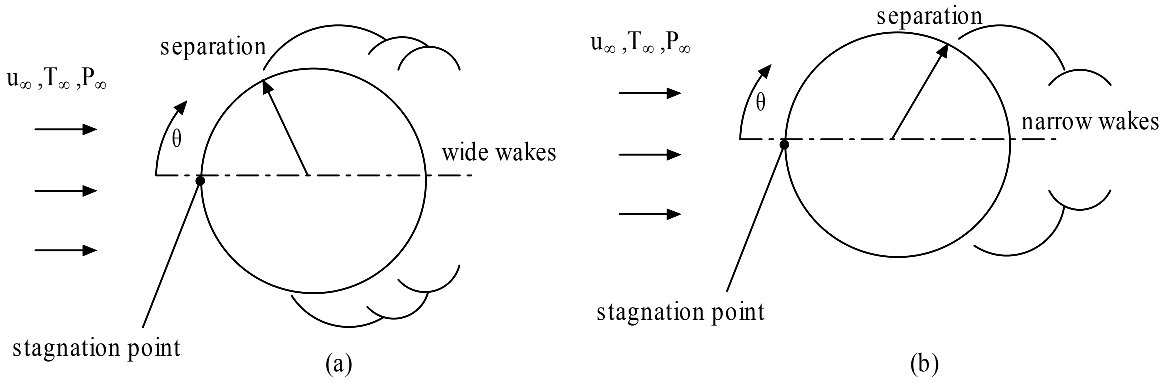


Figure 9.2 Flow over cylinder

Because of the complexity of the flow patterns, only correlations for the average heat transfer coefficients have been developed.

9.3.1 Cylinders

The empirical relation represented by Hilpert given below in Equation 9.13 is widely used, where the constants c, m is given in Table 9.1, all properties are evaluated at film temperature T_f.

$$\overline{Nu}_D = c\,\mathrm{Re}_D^m\,\mathrm{Pr}^{1/3} \tag{9.13}$$

Table 9.1 Constants of Equation 9.13 at different Reynolds numbers

Re_D	c	m
0.4-4	0.989	0.33
4-40	0.911	0.385
40-4000	0.683	0.466
4000-40,000	0.193	0.618
40,000-400,000	0.027	0.805

Other correlation has been suggested for circular cylinder. This correlation is represented by Zhukauskas given below in Equation 9.14 , where the constants c ,m are given in Table 9.2 , all properties are evaluated at Free stream temperature T_∞, except Pr_S which is evaluated at T_S which is used in limited Prandtl number $0.7 < Pr < 500$

$$\overline{Nu}_D = c\,\mathrm{Re}_D^m\,\mathrm{Pr}^n\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_S}\right)^{1/4} \tag{9.14}$$

If $Pr \leq 10$, n= 0.37
If $Pr > 10$, n= 0.36

Table 9.2 Constants of Equation 9.14 at different Reynolds numbers

Re_D	c	m
1-40	0.75	0.4
40-1000	0.51	0.5
1000-2 $x10^5$	0.26	0.6
2 $x10^5$-10^6	0.076	0.7

For entire ranges of Re_D as well as the wide ranges of Prandtl numbers, the following correlations proposed by Churchill and Bernstein (1977): $Re_D\,Pr > 0.2$. Evaluate properties at film temperature T_f:

Re □400, 000

$$\overline{Nu}_D = 0.3 + \frac{0.62\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+\left(\frac{0.4}{\mathrm{Pr}}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}_D}{282{,}000}\right)^{5/8}\right]^{4/5} \tag{9.15}$$

10, 000□Re□ 400, 000

$$\overline{Nu}_D = 0.3 + \frac{0.62\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+\left(\frac{0.4}{\mathrm{Pr}}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}_D}{282{,}000}\right)^{1/2}\right] \qquad (9.16)$$

$\mathrm{Re} \le 10{,}000$

$$\overline{Nu}_D = 0.3 + \frac{0.62\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+\left(\frac{0.4}{\mathrm{Pr}}\right)^{2/3}\right]^{1/4}} \qquad (9.17)$$

For flow of liquid metals, use the following correlation suggested by Ishiguro et al. (1979):
$1 \le \mathrm{Re}_d\,\mathrm{Pr} \le 100$

$$\overline{Nu}_d = 1.125(\mathrm{Re}_D\,\mathrm{Pr})^{0.413} \qquad (9.18)$$

9.3.2 Spheres

Boundary layer effects with flow over circular cylinders are much like the flow over spheres. The following two correlations are explicitly used for flows over spheres,

1. Whitaker (1972): All properties at T_∞ except μ_s at T_s

$3.5 \le \mathrm{Re}_d \le 76{,}000 \qquad 0.71 \le \mathrm{Pr} \le 380 \qquad 1 \le \mu/\mu_s \le 3.2$

$$\overline{Nu}_D = 2 + \left(0.4\,\mathrm{Re}_D^{1/2} + 0.06\,\mathrm{Re}_D^{2/3}\right)\mathrm{Pr}^{2/5}\left(\frac{\mu}{\mu_s}\right)^{1/4} \qquad (9.20)$$

2. Achenbach (1978): All properties at film temperature

$100 \le \mathrm{Re}_d \le 2\times10^5 \qquad \mathrm{Pr} = 0.71$

$$\overline{Nu}_D = 2 + \left(0.25\,\mathrm{Re}_D + 3x10^{-4}\,\mathrm{Re}_D^{8/5}\right)^{1/2} \qquad (9.21)$$

$4\times10^5 \le \mathrm{Re}_d \le 5\times10^6 \qquad \mathrm{Pr} = 0.71$

$$\overline{Nu}_D = 430 + 5x10^{-3}\,\mathrm{Re}_D + 0.25x10^{-9}\,\mathrm{Re}_D^2 - 3.1x10^{-17}\,\mathrm{Re}_D^3 \qquad (9.22)$$

For Liquid Metals convective flow, experimental results for liquid sodium, Witte (1968) proposed,

$3.6\times10^4 \le \mathrm{Re}_d \le 1.5\times10^5$

$$\overline{Nu}_D = 2 + 0.386(\mathrm{Re}_D\,\mathrm{Pr})^{1/2} \qquad (9.23)$$

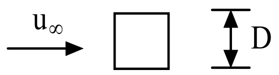
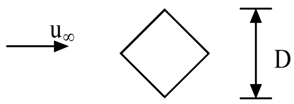
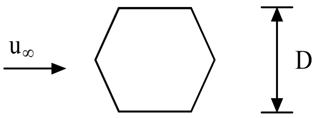
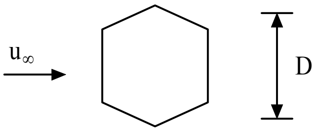
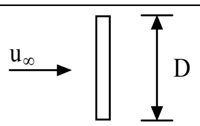
9.3.3 Other Geometries

For geometries other than cylinders and spheres, use Equation 9.24 with the characteristic dimensions and values of the constants given in the Table 9.3 for different geometries, all properties are evaluated at film temperature T_f.

$$\overline{Nu}_D = c\,\mathrm{Re}_D^m\,\mathrm{Pr}^{1/3} \qquad (9.24)$$

Equation 9.24 is based on experimental data done for gases.
Its use can be extended to fluids with moderate Prandtl numbers by multiplying Equation 9.24 by 1.11.

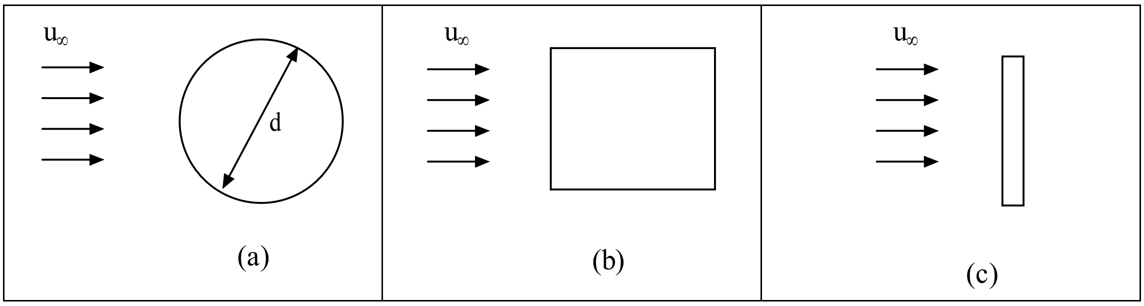
Table 9.3 Constants for Equation 9.24 for non circular cylinders external flow

Geometry	Re_D	C	m
	5×10^3-10^5	0.102	0.675
	5×10^3-10^5	0.246	0.588
	5×10^3-10^5	0.153	0.638
	5×10^3-1.95×10^4 1.95×10^4-10^5	0.16 0.0385	0.638 0.782
	4×10^3-1.5×10^4	0.228	0.731

Example 9.4: Atmospheric air at 25 °C flowing at velocity of 15 m/s. over the following surfaces, each at 75 °C. Calculate the rate of heat transfer per unit length for each arrangement.

a) A circular cylinder of 20 mm diameter ,
b) A square cylinder of 20 mm length on a side
c) A vertical plate of 20 mm height

Schematic:



Solution:

From the film temperature

$$T_f = (25+75)/2 = 50\ ^\circ C$$

Air properties are:

$\nu = 1.8\times10^{-5}$ m/s.

k = 0.028W/m. °C

Pr= 0.70378

Calculation of Reynolds number for all cases have the same characteristic length=20mm

$$Re_D = u_\infty L / \nu = 15 \times 0.02 / 1.8\times10^{-5} = 16666.667$$

By using Equation 9.13 for all cases

$$\overline{Nu}_D = c\, Re_D^m\, Pr^{1/3}$$

Case (a) a circular cylinder
From Table 9.1
C = 0.193 and m= 0.618

$$\overline{Nu}_D = 0.193(16666.667)^{0.618}(0.70378)^{1/3}$$
$$= 69.79$$

The average heat transfer coefficient is

$$\bar{h}_D = \frac{\overline{Nu}_D k}{D} = \frac{69.79x0.028}{0.02} = 97.7 \quad W/m^2.^oC$$

The total heat transfer per unit length is

$$q^/ = 97.7\ (\pi \text{ x } 0.02)50 = 306.9 \text{ W/m}$$

Case (b) a square cylinder
From Table 9.3
C = 0.102 and m= 0.675

$$\overline{Nu}_D = 0.102(16666.667)^{0.675}(0.70378)^{1/3}$$
$$= 64.19$$

The average heat transfer coefficient is

$$\bar{h}_D = \frac{\overline{Nu}_D k}{D} = \frac{64.19x0.028}{0.02} = 89.87 \quad W/m^2.^oC$$

The total heat transfer per unit length is

$$q^/ = 89.87\ (4 \text{ x } 0.02)50 = 359.48 \text{ W/m}$$

Case (c) a vertical plate
From Table 9.3
C = 0.228 and m= 0.731

$$\overline{Nu}_D = 0.228(16666.667)^{0.731}(0.70378)^{1/3}$$
$$= 247.316$$

The average heat transfer coefficient is

$$\bar{h}_D = \frac{\overline{Nu}_D k}{D} = \frac{247.316x0.028}{0.02} = 346.24 \quad W/m^2.^oC$$

The total heat transfer per unit length is

$$q^/ = 346.24 \text{ x } (2 \text{ x } 0.02) \text{ x } 50 = 692.48 \text{ W/m}$$

9.4 Heat Transfer across Tube Banks (Heat Exchangers)

Heat transfer through a bank (or bundle) of tubes has several applications in industry as heat exchanger which can be used in many applications.
When tube banks are used in heat exchangers, two arrangements of the tubes are considered aligned and staggered as shown in Figure 9.3.
If the spacing between the tubes is very much greater than the diameter of the tubes, correlations for single tubes can be used. Correlations for flow over tube banks when the spacing between tubes in a row and a column are not much greater than the diameter of the tubes have been developed for use in heat-exchanger applications will be discussed as follows.

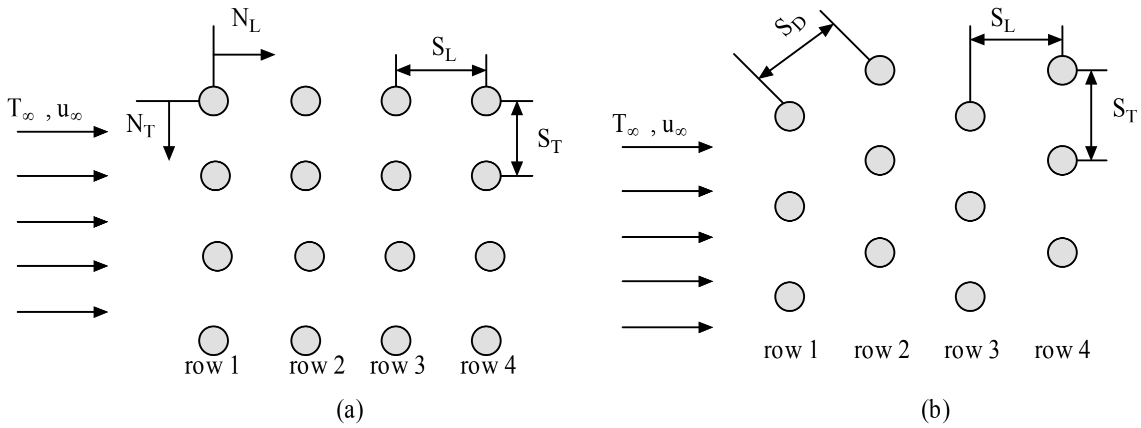


Figure 9.3 Arrangements of the tubes (a) In-line arrangement, (b) staggered arrangement

For the average convective heat transfer coefficient with tubes at uniform surface temperature, experimental results carried by Zukauskas (1987) recommended the following correlation:

$$\overline{Nu}_D = c(a/b)^P \, \mathrm{Re}_D^m \, \mathrm{Pr}^n (\mathrm{Pr}/\mathrm{Pr}_s)^{0.25} \qquad (9.25)$$

$a = S_T/D$; $b = S_L/D$.
S_T = Transverse pitch
S_L = Lateral pitch
D = tube diameter

All properties are evaluated at the arithmetic mean of the inlet and exit temperatures of the fluid $(T_{\infty i} + T_{\infty o})/2$, except Pr_S which is evaluated at the surface temperature T_S. The values of the constants c, p, m, and n are given in Table 9.4 for in-line arrangement and in Table 9.5 for staggered arrangement. The maximum average velocity between tubes is used to calculate Re_D. The maximum velocity can be calculated as follows:

For in-line arrangement:

$$u_{max} = u_\infty \left(\frac{S_T}{S_T - D} \right)$$

For Staggered arrangement: If

$$S_D > \frac{S_T + D}{2}$$

$$\therefore u_{max} = \frac{S_T}{S_T - D} u_\infty$$

If not

$$\therefore u_{max} = \frac{0.5 S_T}{(S_D - D)} u_\infty$$

Where

$$S_D = \left[S_L^2 + \left(\frac{S_T}{2} \right)^2 \right]^{1/2}$$

Table 9.4 In-Line arrangement values of constants in Equation 9.25 (p = 0 in all cases)

Re_D	C	m	n
1-100	0.9	0.4	0.36
100-1000	0.52	0.5	0.36
$1000\text{-}2\times10^5$	0.27	0.63	0.36
$2\times10^5\text{-}2\times10^6$	0.033	0.8	0.4

Table 9.5 Staggered arrangement values of constants in Equation 9.25

Re_D	c	P	m	n
1-500	1.04	0	0.4	0.36
500-1000	0.71	0	0.5	0.36
1000-2x10^5	0.35	0.2	0.6	0.36
2x10^5-2x10^6	0.031	0.2	0.8	0.36

The temperature of the fluid varies in the direction of flow, and, therefore, the value of the convective heat transfer coefficient (which depends on the temperature-dependent properties of the fluid) also varies in the direction of flow.
It is a common practice to compute the total heat transfer rate with the assumption of uniform convective heat transfer coefficient. With such an assumption of uniform convective heat transfer coefficient, uniform surface temperature and constant specific heat, the heat transfer rate to the fluid is related by the heat balance.

$$q = m^{\bullet} C_P (T_{\infty o} - T_{\infty i}) = \bar{h} A \left[T_S - \left(\frac{T_{\infty o} + T_{\infty i}}{2} \right) \right] \qquad (9.26)$$

$T_{\infty i}$ = inlet free stream temperature
$T_{\infty o}$ = outlet free stream temperature
$m^{\bullet}$ = out side tubes gaseous flow rate
= $\rho_{\infty i}(N_T S_T L)u_{\infty}$
A = Total heat transfer area
= $N_T N_L \pi DL$
N_T = number of tubes in transverse direction
N_L = number of tubes in lateral direction
L = length of tube per pass.

Pressure drop across tube banks:
Pressure drop is a significant factor, as it determines the power required to move the fluid across bank. The pressure drop for flow gases over a bank may be calculated with Equation 9.27.

$$\Delta p = \frac{2 f^{/} G_{\max}^{2} N_T}{\rho} \left(\frac{\mu_S}{\mu} \right)^{0.14} \qquad (9.27)$$

Δp = pressure drop in pascals.
$f^{/}$ = friction factor is given by Jacob
G_{max}= mass velocity at minimum flow rate, kg/m^2.s
ρ = density evaluated at free stream conditions, kg/m^3
N_T = number of transverse rows.
μ_S = fluid viscosity evaluated at surface temperature.
μ = average free stream viscosity.

For in-line:

$$f' = \left[0.044 + \frac{0.08 S_L / D}{\left(\frac{S_T - D}{D} \right)^{0.43 + 1.13 D / S_L}} \right] \mathrm{Re}_{\max}^{-0.15} \tag{9.28}$$

For staggered:

$$f' = \left[0.25 + \frac{0.118}{\left(\frac{S_T - D}{D} \right)^{1.08}} \right] \mathrm{Re}_{\max}^{-0.16} \tag{9.29}$$

Example 9.5: A heat exchanger with aligned tubes is used to heat 40 kg/sec of atmospheric air from 10 to 50 °C with the tube surfaces maintained at 100 °C. Details of the heat exchanger are
Diameter of tubes 25 mm
Number of columns (N_T) 20
Length of each tube 3 m
$S_L = S_T$ □75 mm
-Determine the number of rows (N_L) required

Solution:
Properties of atmospheric air at average air temperature = $(T_{\infty i} + T_{\infty o})/2$ = 30 □C.

ρ = 1.151 kg/ m^3
C_P= 1007 J/ kg.°C
k = 0.0265 W/ m.°C
μ = 186x10^{-7} N.s/ m^2
Pr = 0.7066

At surface temperature T_s=100 °C
Pr_s = 0.6954

At inlet free stream temperature $T_{\infty i}$=10 °C
$\rho_{\infty i}$ = 1.24 kg/ m^3

To find u_∞

$$\dot{m} = \rho_{\infty i}(N_T S_T L) u_\infty$$
$$40 = 1.24(20 \mathrm{x} 0.075 \mathrm{x} 3)\, u_\infty$$
$$u_\infty = 7.168 \text{ m/s}$$

For in-line arrangement:

$$u_{\max} = u_\infty \left(\frac{S_T}{S_T - D} \right)$$
$$= 7.168 \times 0.075\ (0.075 - 0.025)$$

$$= 10.752 \text{ m/ s}$$

Reynolds number based on maximum velocity

$$Re_D = \rho u_{max} D/\mu = 1.151\times10.752\times0.025/(186\times10^{-7})$$
$$= 16633.8$$

From Table 9.4

$$p = 0$$
$$C = 0.27$$
$$m = 0.63$$
$$n = 0.36$$

From Equation 9.25

$$\overline{Nu}_D = 0.27\,(16633.8)^{0.63}\,(0.7066)^{0.36}(0.7066/0.6954)^{0.25} = 109.15$$
$$= \bar{h}\,D/k$$

The average heat transfer coefficient is

$$\bar{h} = 109.15\text{x}0.0265/0.025 = 115.699 \text{ W/m}^2.\text{k}$$

From Equation 9.26.The Total heat transfer is

$$q = m^{\bullet} C_P (T_{\infty o} - T_{\infty i}) = \bar{h}A\left[T_S - \left(\frac{T_{\infty o} + T_{\infty i}}{2}\right)\right]$$
$$40\text{x}1007\,(50 - 10) = 115.699A\,[100-30]$$

Total heat transfer area

$$A = 198.94 \text{ m}^2$$

Number of rows (N_L)

$$A = N_T N_L \pi D L$$
$$198.94 = 20N_L\,(\pi \text{ x } 0.025 \text{ x } 3)$$
$$N_L = 43 \text{ rows}$$

9.5 Heat Transfer with Jet Impingement

The heat transfer with jet impingement on a heated (or cooled) surface results in high heat transfer rates, and is used in a wide range of applications such as cooling of electronic equipments. Usually, the jets are circular with round nozzle of diameter d, or rectangular with width w. They may be used individually or in an array. The jets may impinge normally to the heated surface as shown in Figure 9.4 or at an angle.

The liquid coolant under pressure inside a chamber is allowed to pass through a jet and directly into the heated surface, there are two modes of operation possible with liquid jet impingement, namely single phase cooling, and two phases cooling. And the jet may be free or submerged.

If there is no parallel solid surface close to the heated surface, the jet is said to be free as shown in Figure 9.4, while Jets may be submerged if the cavity is completely filled with the fluid (from the nozzle into a heated surface). In this lecture only single, free jets (round or rectangular) impinging normally to the heated surface are considered.

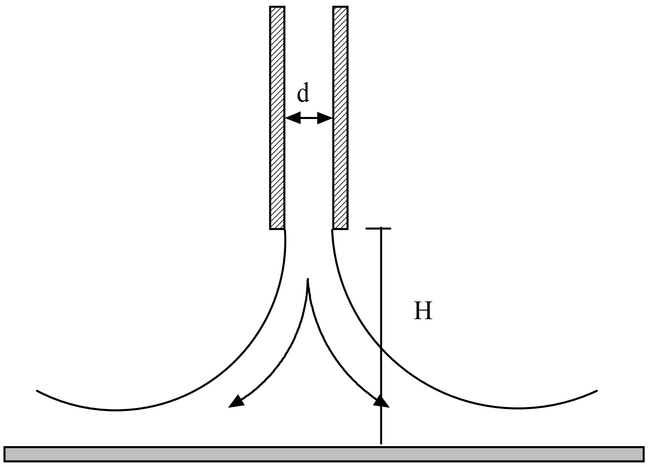


Figure 9.4 Jet impinge normally to the heated surface

Cooling analysis:

Free single phase jet impingement cooling is affected with many variables such as:

- Jet diameter (d)
- Fluid velocity (v)
- Jet to heated surface distance (H)
- Size of heated surface area (L x L)
- Coolant properties

The average heat transfer coefficient correlation is given by Jigi and Dagn

$$\overline{Nu}_L = 3.84\,\mathrm{Re}_d^{0.5}\,\mathrm{Pr}^{0.33}\left(0.008\frac{L}{d}+1\right) \tag{9.30}$$

The properties are evaluated at mean film temperature $(T_S + T_\infty)/2$

This correlation is experimented for FC-77 and water and also valid for 3< H/d <15

- $3 < H/d < 15$
- d = 0.508 to 1.016 mm
- v < 15 m/s
- small surface dimensions L<12.7 mm (microelectronic devices)

Example 9.6: A single phase free jet impingement nozzle is placed in the center of an electronic heated surface 12x12 mm^2 and the heated surface is placed at 4 mm from the jet.

The working medium is FC-77 passing through 1mm tube diameter at a rate of 0.015 kg/s.

To cool the plate, if the supply coolant is at 25 ºC and the heat load is 20 W

Determine the average heat transfer coefficient and the surface temperature of the heated surface.

Solution:

To get the properties of the FC-77 we need to assume the surface temperature: let the surface temperature equal 45 ºC as a first approximation.

$$T_f = (45 + 25)/2 = 35\,^{\circ}C$$

From FC-77 property tables at 35 ºC:

$\rho = 1746$ kg/m^3

$\mu = 1.198\times10^{-3}$ N.s/m^2

k = 0.0623 W/m.ºC

Pr= 20.3

We must check on H/d ratio:

$$H/d = 4/1$$

Calculation of Reynolds number

$$Re = \rho v d / \mu = 4m' / \pi d \mu$$
$$= 4\times0.015 / \pi \times1\times10^{-3}\times1.198\times10^{-3}$$
$$= 15942$$

From Equation 9.30

$$\overline{Nu}_L = 3.84\,Re_d^{0.5}\,Pr^{0.33}\left(0.008\frac{L}{d}+1\right)$$
$$= 3.84\,(15942)^{0.5}\,(20.3)^{0.33}(0.008\times12/1+1)$$
$$= 1435.12$$

Knowing that $\overline{Nu}_l = \dfrac{\bar{h}L}{k}$

And so the average heat transfer coefficient is:

$$\bar{h} = (1435.12\times0.0623)/\,(12\times10^{-3})$$
$$= 7450.656\ W/m^2.k$$

But since $q = \bar{h}\,A\,\Delta T$

Then the temperature difference is:

$$\Delta T = 20/\,(7450.656\times12\times12\times10^{-6})$$
$$= 18.64\ ^{o}C$$

Therefore the surface temperature is:

$$T_S = 43.64\ ^{o}C$$

For more accuracy we may make another trial at new film temperature and reach more accurate results.

9.5.1 Case Study

The use of impinging air jets is proposed as a means of effectively cooling high-power logic chips in a computer. However, before the technique can be implemented, the convection coefficient associated with jet impingement on a chip surface must be known. Design an experiment that could be used to determine convection coefficients associated with air jet impingement on a chip measuring approximately 10 mm by 10 mm on a side.

9.6 Internal Flows (Inside Tubes or Ducts)

The heat transfer to (or from) a fluid flowing inside a tube or duct used in a modern instrument and equipment such as laser coolant lines ,compact heat exchanger , and electronics cooling(heat pipe method). Only heat transfer to or from a single-phase fluid is considered.

The fluid flow may be laminar or turbulent, the flow is laminar if the Reynolds number ($\rho u_m D_H/\mu$) is less than 2300 ($Re \le 2300$), based on the tube hydraulic diameter ($D_H = 4A / P$) where A, P is the cross sectional area and wetted perimeter respectively and u_m is average velocity over the tube cross section. Also the hydraulic diameter should be used in calculating Nusselt number. And If the Reynolds number is greater than 2300 the flow is turbulent ($Re > 2300$).

9.6.1 Fully Developed Velocity Profiles

When a fluid enters a tube from a large reservoir, the velocity profile at the entrance is almost uniform because the flow at the entrance is almost inviscid while the effect of the viscosity increase with the length of tube (in x-direction) which have an effect on the shape of the velocity profile as shown in Figure 9.5.

At some location downstream of the pipe inlet, the boundary layer thickness (δ) reaches its maximum possible value, which is equal to the radius of the tube at this point and the velocity profile does not change.
The distance from the entrance to this point is called fully developed region or entrance region, and it is expressed as the fully developing length (L_D), this length depends on the flow which may be laminar or turbulent.
Through the developing length (L_D) the heat transfer coefficient is not constant and after the developing length the heat transfer coefficient is nearly constant as shown in Figure 9.5.

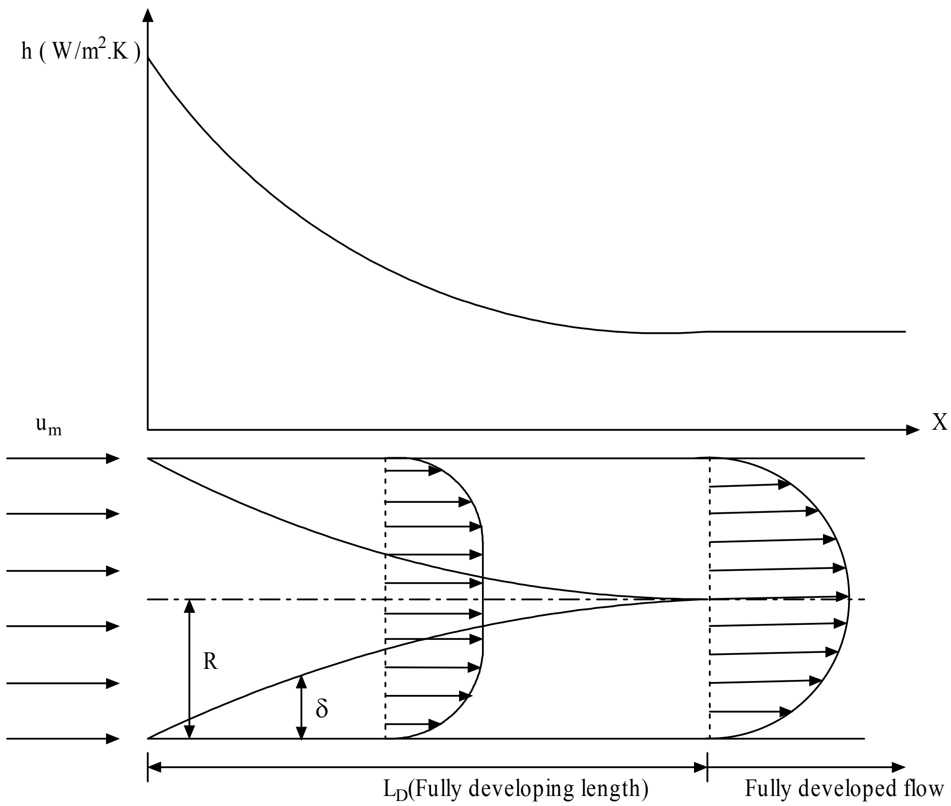


Figure 9.5 Flow inside the pipe

The fully developing length is given by:

$L_D/D = 0.05Re\ Pr$ -For laminar flow

And

$L_D/D = 10$ -For turbulent flow

9.6.2 Heat Transfer Correlations

Through the entrance region and after this region (fully developed flow) both regions have different heat transfer correlations, given in Table 9.6.

Table 9.6 Summery for forced convection heat transfer correlations - internal flow

Correlations		Conditions
Sieder and Tate (1936) $\overline{Nu}_D = 1.86\left(\frac{Re_D\,Pr}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_S}\right)^{0.14}$	(9.31)	- Laminar , entrance region - uniform surface temperature - L/D < $\frac{Re_D\,Pr}{8}\left(\frac{\mu}{\mu_S}\right)^{0.42}$ - 0.48 < Pr <16,700 - 0.0044 < μ/μ_S < 9.75
Nu_D =3.66	(9.32)	- Laminar , fully developed - Uniform surface temperature - $Pr \geq 0.6$
Nu_D = 4.36	(9.33)	- Laminar , fully developed - Uniform Heat Flux - $Pr \geq 0.6$
Dittus–Boelter (1930) $Nu_D = 0.023\,Re_d^{4/5}\,Pr^n$	(9.34)	- Turbulent flow, fully developed - $0.7 \leq Pr \leq 160$, $L/D \geq 10$ - n = 0.4 for heating ($T_s > T_m$) and - n = 0.3 for cooling ($T_s < T_m$).
Sleicher and Rouse (1976) $Nu_{D,m} = 4.8 + 0.0156\,Re_{D,f}^{0.85}\,Pr_S^{0.93}$ $Nu_{D,m} = 6.3 + 0.0167\,Re_{D,f}^{0.85}\,Pr_S^{0.93}$	(9.35) (9.36)	- For liquid metals , Pr «1 - Uniform surface temperature - For liquid metals , Pr «1 - Uniform heat flux

Properties of the fluid for each equation:
For Equation 9.31 at the arithmetic mean of the inlet and exit temperatures $(T_{mi}+T_{mo})/2$
For Equation 9.32, Equation 9.33 Equation 9.34 and at the mean temperature T_m
For Equation 9.35 and Equation 9.36 the subscripts m, f, and s indicate that the variables are to be evaluated at the mean temperature T_m, film temperature T_f (arithmetic mean of the mean temperature and surface temperatures $(T_S + T_m)/2$, and surface temperature, respectively.

9.6.3 Variation of Fluid Temperature(T_m) in a Tube

By taking a differential control volume as shown in Figure 9.6 considering that the fluid enters the tube at T_{mi} and exits at T_{mo} with constant flow rate m', convection heat transfer occurring at the inner surface (h), heat addition by convection q, neglecting the conduction in the axial direction, and assuming no work done by the fluid. Then applying heat balance on the control volume we can obtain an equation relating the surface temperature at any point.

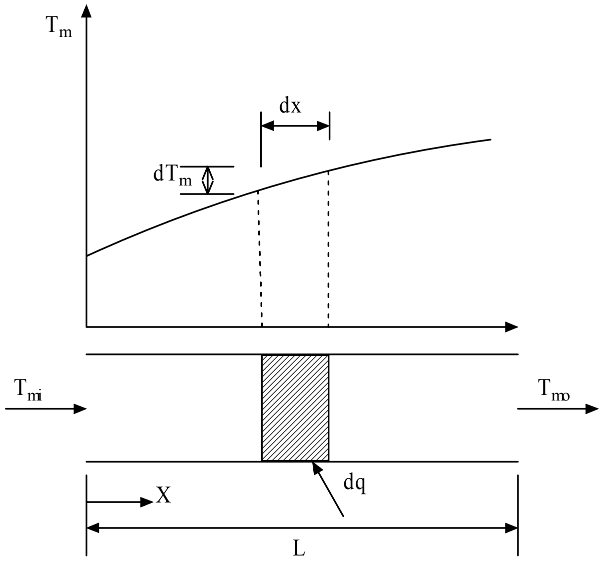


Figure9.6 differential control volume inside tube

Case 1:

- At uniform surface temperature T_S = constant

$$dq = m^{'}\, C_p\, dT_m \quad (9.37)$$

$$= h\,(P_w dx)\,(T_S - T_m) \quad (9.38)$$

Where P_w = out side tube perimeter

By equating the both Equations 9.37, 9.38

$$\frac{dT_m}{dx} = \frac{hP_w}{m^{'}C_p}(T_S - T_m) \quad (9.39)$$

$$T_S - T_m = \Delta T$$

$$\frac{dT_m}{dx} = -\frac{d\Delta T}{dx}$$

$$-\frac{d\Delta T}{dx} = \frac{hP_w}{m^{'}C_p}(\Delta T)$$

$$-\frac{d\Delta T}{\Delta T} = \frac{hP_w}{m^{'}C_p}dx \quad (9.40)$$

By integration of Equation 9.40 as shown bellow:

$$\int_{\Delta T_i}^{\Delta T_X} \frac{d\Delta T}{\Delta T} = \int_0^x \frac{hP_w}{m^{'}C_p}dx$$

$$\int_{\Delta T_i}^{\Delta T_X} \frac{d\Delta T}{\Delta T} = \frac{xP_w}{m^{'}C_p}\left(\frac{1}{x}\int_0^x h\,dx\right)$$

$$-\ln\left(\frac{\Delta T_x}{\Delta T_i}\right) = \frac{P_w \bar{h}}{m^{\cdot} C_p} x$$

$$\frac{\Delta T_x}{\Delta T_i} = e^{-\left(\frac{P_w \bar{h}}{m^{\cdot} C_p} x\right)}$$

At x = L

$$\frac{\Delta T_o}{\Delta T_i} = e^{-\left(\frac{p_w \bar{h}}{m^{\cdot} C_p} L\right)} \tag{9.41}$$

So that the temperature distribution along the tube is as shown in Figure 9.7

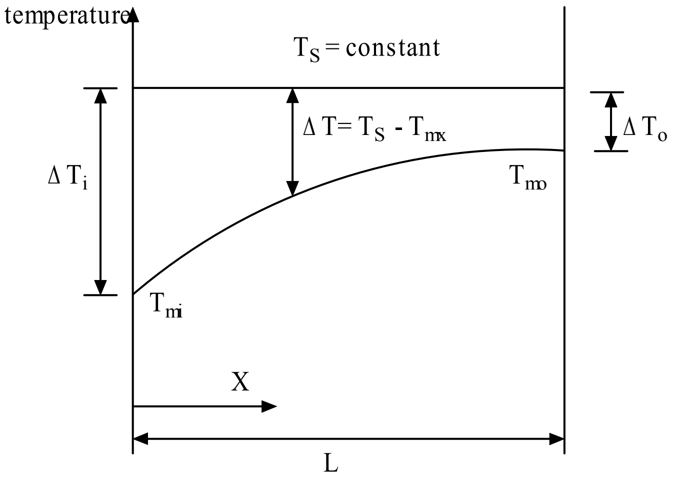


Figure 9.7 Temperature distributions along the tube at uniform surface temperature

The total heat transfer is:

$$q_t = m^{\cdot} C_p (T_{mo} - T_{mi}) \tag{9.42}$$

But also

T_{mo}- T_{mi} = $\Delta T_i - \Delta T_o$

So that the Equation 9.42 can be written as follows

$$q_t = m^{\cdot} C_p (\Delta T_i - \Delta T_o) \tag{9.43}$$

Also for total heat transfer by average heat transfer coefficient

$$q_t = \bar{h} L P_w (T_S - T_m)_{average} \tag{9.44}$$

From Equations 9.43, 9.44 produce:

$$m^{\cdot} C_p (\Delta T_i - \Delta T_o) = \bar{h} L P_w (\Delta T)_{average}$$

$$\Delta T_{average} = (\Delta T_i - \Delta T_o) \frac{m^{\cdot} C_p}{\bar{h} L P_w} \tag{9.45}$$

Where $\Delta T_{average} = (T_S - T_m)_{average}$

Substitute the Equation 9.41 in Equation 9.45 produces:

$$\Delta T_{average} = \frac{\Delta T_i - \Delta T_o}{\ln\left(\frac{\Delta T_i}{\Delta T_o}\right)} \tag{9.46}$$

$\Delta T_{average}$ called logarithmic mean temperature difference (LMTD)

Case 2:

- At uniform heat flux $q^{//}$ = constant

From Equation 9.37

$$dq = m^{'}\,C_p\,dT_m$$

And

$$dq = q^{//}\,P_w dx \tag{9.47}$$

Combining both Equations 9.37, 9.47 produces:

$$\frac{dT_m}{dx} = \frac{q^{//}P_w}{m^{'}C_p} \tag{9.48}$$

By integration:

$$\int_{T_{mi}}^{T_{mx}} dT_m = \frac{q^{//}P_w}{m^{'}C_p}\int_0^x dx$$

$$T_{mx} - T_{mi} = \frac{q^{//}P_w}{m^{'}C_p}\,x \tag{9.49}$$

And

$$q^{//} = h\,(T_S - T_m)$$

$$T_S - T_m = q^{//} / h \approx \text{constant along the tube length} \tag{9.50}$$

From the Equations 9.49, 9.50 the temperature distribution as shown in Figure 9.8.

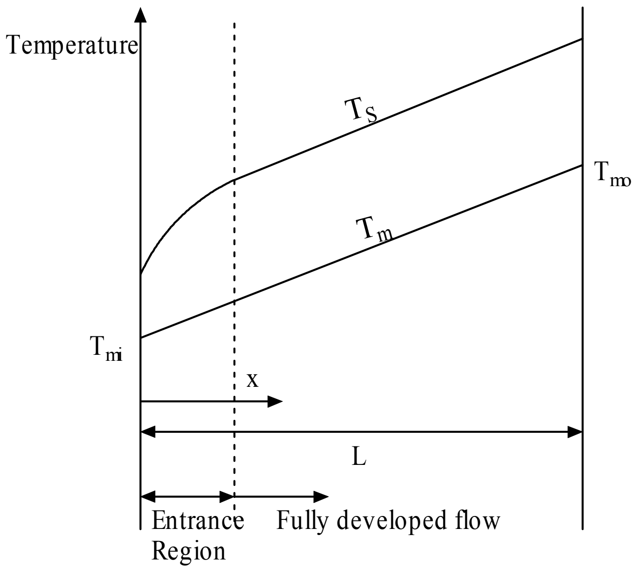


Figure 9.8 Temperature distributions along the tube at uniform heat flux

For along the tube length

$$T_{mo} = T_{mi} + \frac{q^{//}P_w}{m^{'}C_p}\,L$$