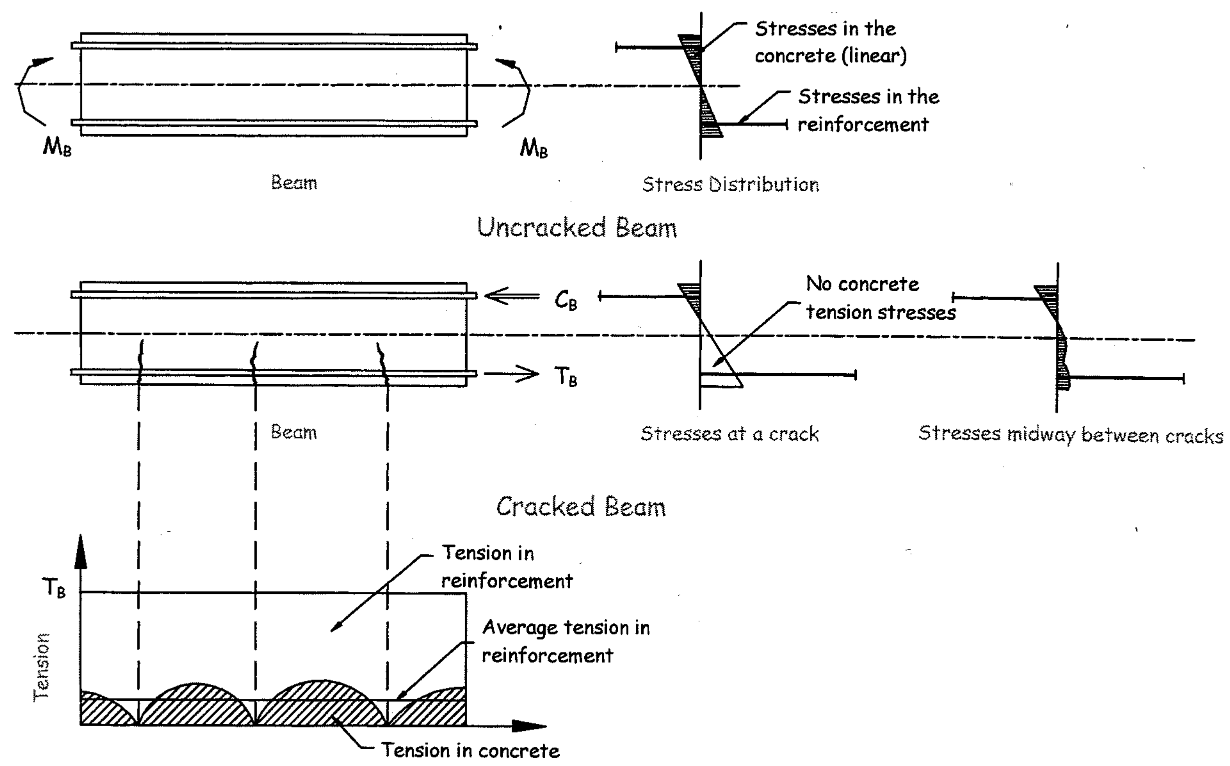
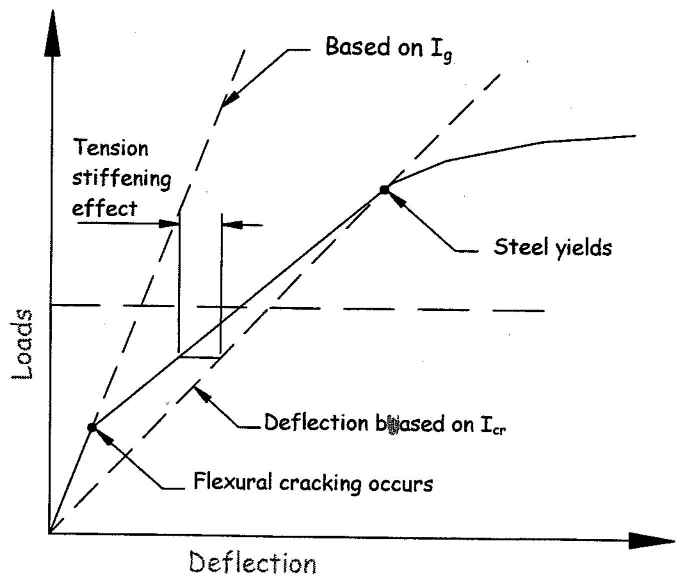


Tension Stiffening

Tension carried by the concrete between the cracks reduces the average tensile strain in the reinforcement, effectively making the section stiffer.

When the section is cracked, the concrete continues to carry tension in the regions between the cracks. This reduces the average tensile strain in the reinforcement, resulting in a stiffer section. Cracked section response is typically calculated on the basis of the transformed cracked section for calculating stresses. This neglects tension stiffening effects. Where these are important, for example in calculating deflections, a nominal allowance is made. Note that the flexural stiffness of a cracked beam is typically about half that of the uncracked equivalent. Where tension stiffening is important, as for example in calculating deflections, nominal allowance is made for this effect.



12.6 Cracked Transformed Sections

When determining member stresses, it is necessary to ensure member force equilibrium:

$$C_c + C_s = T \text{ or } f_c A_c + f_s' A_s' = f_s A_s$$

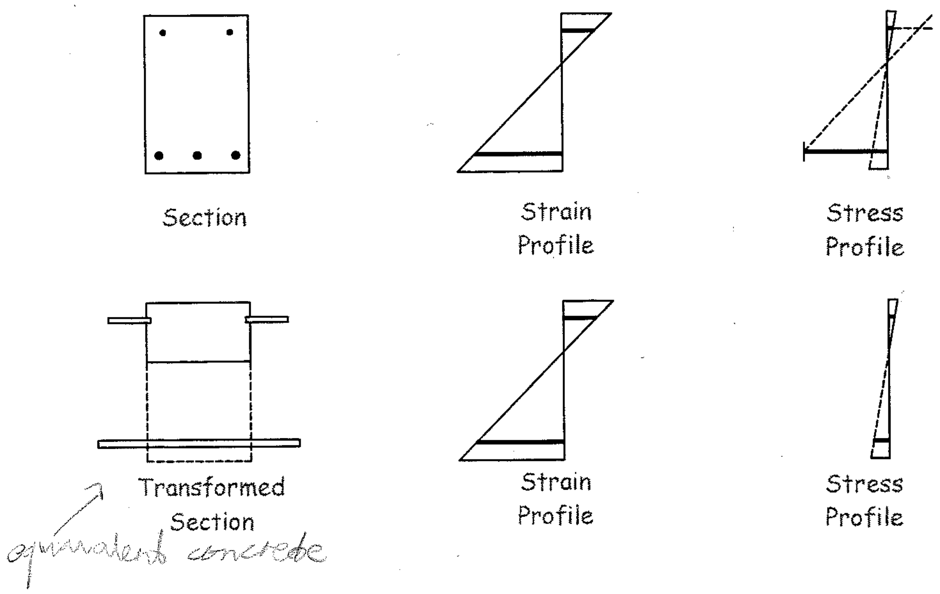
Working from a linear strain assumption for both the concrete and the reinforcement, and recognising that strains will remain in the elastic regime, the stress in each component can be related to the strain using:-

$$f_c = E_c \varepsilon_c \text{ and}$$

Consequently the stress profile is not linear, as shown below. By transforming the reinforcement to concrete by the modular ratio:

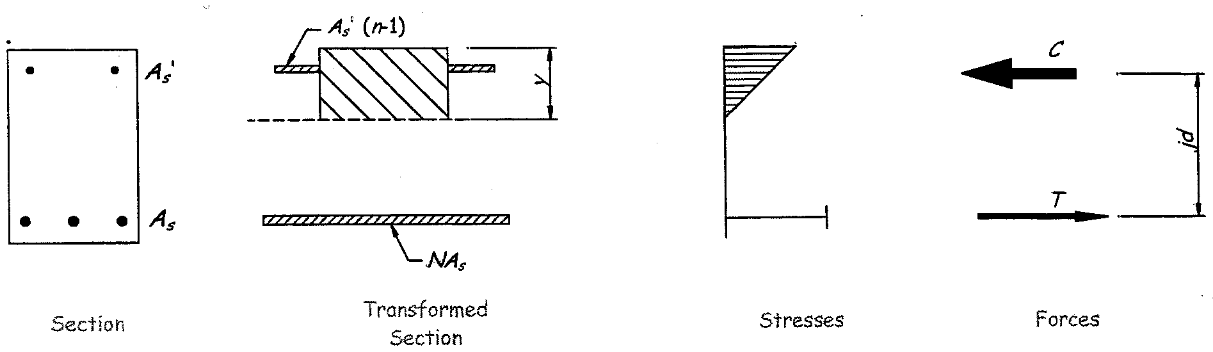
$$n = \frac{E_s}{E_c}$$

an equivalent concrete section is obtained, having both a linear strain profile and a linear stress profile. Using similar triangles, it is then relatively simple to develop equations that have the neutral axis depth as the only unknown. Note that this implies that at the serviceability limit state the neutral axis depth is constant, regardless of the stresses acting upon the section.



12.6.1 Transformed Section Analysis Procedure

When concrete is subjected to flexural tension it is assumed to crack, and support no tension forces. Consequently it is disregarded in the calculation.



The method used is:

(i) Find the neutral axis position by:

(a) 1st moment of areas (which is ok if there is no axial load)

or (b) trial and error in balancing forces

(ii) Find I of the transformed section

(iii) To find stresses in the section, two different methods may be used:

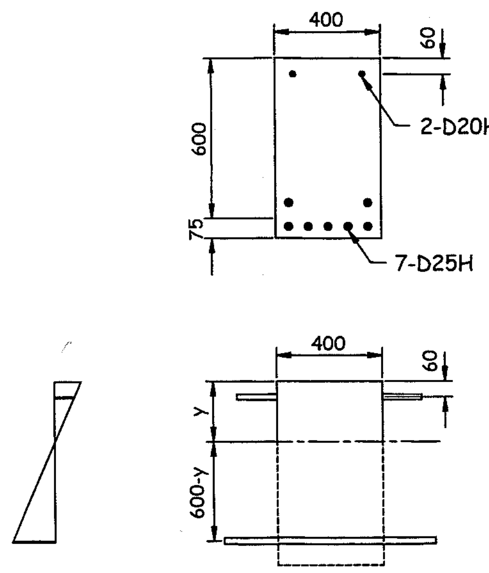
(a) use $f = \frac{My}{I}$ allowing for modular ratio n

prone to errors

(b) find jd from the stress distribution, then find $T = \frac{M}{jd}$ etc

Example

If f'_c = 25 MPa and the serviceability bending moment is 330 kNm, find the stresses in the concrete and the tension reinforcement.



$E_c = 3320\sqrt{25} + 6900 =$ 23500 MPa

$E_s = 200000$ MPa

$n = \frac{E_s}{E_c} = \frac{200000}{23500} =$ 8.51

$A_s' = 2\text{-D20H} = 2 \times 314 =$ 628 mm²

$(n-1)A_s' =$ 4719 mm²

$A_s = 7\text{-D25H} = 7 \times 491 =$ 3436 mm²

$nA_s =$ 29241 mm²

For force equilibrium the sum of the first moments of transformed area about the neutral axis = 0.

Alternatively, recognising that stress is linearly related to the distance from the neutral axis:

$$f_c \propto \frac{y}{2},\ f_s' \propto (y-60),\ f_s \propto (600-y)$$

Force equilibrium may be expressed as:

$$C_c + C_s = T$$

$$\Rightarrow A_c f_c + (n-1)A_s' f_s' = nA_s f_s$$

$$\Rightarrow 400y\frac{y}{2} + 4719\,(y-60) = 29241\,(600-y)$$

$200y^2 + 4719y - 283140 + 29241y - 17544600 = 0$

$200y^2 + 33960y - 1782774 = 0$

The solution to this expression can then be obtain by rearranging to form a quadratic equation, or by finding a solution by trial and error:

Try y = 200 mm	8,000,000 + 660,660 = 8,660,660 (LHS) 11,696,400 (RHS) → Therefore increase y
Try y = 220 mm	9,680,000 + 755,040 = 10,435,040 (LHS) 11,111,580 (RHS) → Therefore increase y
Try y = 225 mm	10,125,000 + 778,635 = 10,903,635 (LHS) 10,965,375 (RHS) → Therefore OK

12.6.2 Cracked Section Stiffness

Once the location of the neutral axis has been determined, the cracked section stiffness can be found using the parallel axis theorem. Note that generally the reinforcement will have little moment of inertia about its local axis, so that only rotation of the reinforcement about a displaced axis needs to be considered.

$$\text{hence, } I_{cr} = \frac{400 \text{ x } 225^3}{12} + 400 \text{ x } 225 \text{ x} \left(\frac{225}{2}\right)^2 + 4719\,(225-60)^2 + 29241\,(600-225)^2$$

$$= 5.76 \times 10^9 \ mm^4$$

Note that I_g of the untransformed section (ignoring reinforcement) is:

$$\frac{bh^3}{12} = \frac{400 \text{ x } (600+75)^3}{12} = 10.25 \times 10^9 \ mm^2$$

$$\text{Therefore,} \quad I_{cr} = \frac{5.76 \text{ x } 10^9}{10.25 \text{ x } 10^9} = 0.56\, I_g$$

Note also that the above calculation has assumed a cracked section, but has not been dependent upon the applied loading.

12.6.3 Determining Stresses

To find *jd*, it is necessary to find the centroid of the compression forces in the concrete and reinforcement. As the section is behaves linearly (constant neutral axis depth and constant *jd*), the forces for any stress in the top fibre can be calculated and these values can be used to find *jd*. This may be done by assuming a compression stress at the extreme compression fibre, or by conducting the calculation treating the extreme compression stress as an unknown.

Assume f = 10 MPa

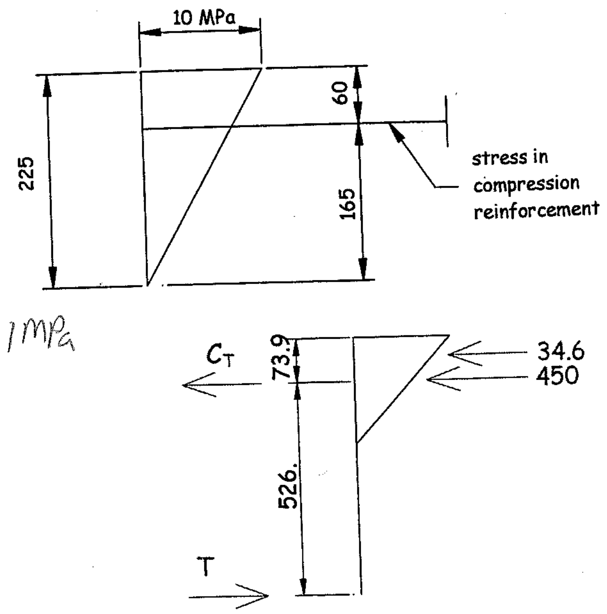
$$C_c = 400 \text{ x } \underbrace{225}_{\textit{depth}} \text{ x } \underbrace{10/2}_{\textit{assumed stress}} = 450 \text{ kN}$$

x = 225/3 = 75.0 mm

Stress in 2-D20H = 165/225 x 10 x 7.51 = 55.1 MPa (n-1)

should be (31.2 kN)

C'_s = 628 x 55.1 = 34163.2 N = 34.6 kN



Location of the centroid of C:

$$\bar{x} = \frac{(34.6 \text{ x } 60 + 450 \text{ x } 75)}{(34.6+450)} = 73.9 \text{ mm}$$

therefore, jd = 600 - 73.9 = 526.1 mm

hence $$T = \frac{M}{jd} = \frac{330}{0.5261} = 627 \text{ kN}$$

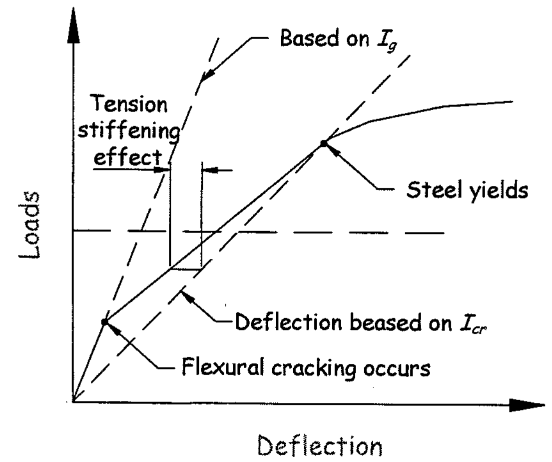
and $$f_s = \frac{T}{A_s} = \frac{627 \text{ x } 1000}{3436} = 182.5 \text{ MPa}$$

$\therefore$ actual $f_c = \frac{627}{1} \times \frac{10}{34.6+450} = 12.9$ MPa   actual $f_s' = 627 \times \frac{55.1}{34.6+450} = 71.3$ MPa

12.7 Deflection Calculations

To calculate deflections an effective moment of inertia, I_e, is used for the beam, or part of the beam, where:

$$I_g \geq I_e \geq I_{cr}$$



The value of I_e varies with the stress level in the beam. It is greater than I_{cr} to allow for tension stiffening effects, as discussed earlier. In the New Zealand code the effective moment of inertia is given by (see 3.3.2.3):

$$I_e = \left[\frac{M_{cr}}{M_a}\right]^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr}$$

actual moment

where: I_{cr} = moment of inertia of cracked transformed section,

I_g = moment of inertia of gross section,

M_{cr} = flexural cracking moment (Note: f_r is not a reliable strength),

$= f_r \dfrac{I_g}{y_t}$

y_t = distance from the neutral axis to the tension fibre,

f_r = modulus of rupture = $0.6\sqrt{f_c'}$ MPa (Cl 3.8.1.3)

M_a = the maximum bending moment in the beam (or part of the beam).